University of Caloocan City

*College of Liberal Arts & Sciences*

**Mathematics Department**

**INTEGRAL CALCULUS: Learning Module No. 5**

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| **Topic** | **METHODS OF INTEGRATION** |
| **Sub-Topic** | **Integration By Parts** |
| **Duration** | 3 hours |
| **Introduction**  Note: This module may contain copyrighted material. The use of which has not been specifically authorized by the copyright owner. This module is for educational purpose only for online instruction and is not used to generate profit. Thus this constitutes a “Fair Use” of the copyrighted material as provided by virtue of Republic Act No. 8293 otherwise known as Intellectual Property Code of the Philippines. | The method of Integration by Parts is specifically helpful when the integrand is a product of two kinds of functions such as the following:   1. Algebraic and Trigonometric   ∫ 𝑥2𝑠𝑖𝑛𝑥 𝑑𝑥   1. Algebraic and Logarithmic   ∫ 𝑥2𝑙𝑛𝑥 𝑑𝑥   1. Algebraic and Exponential   ∫ 𝑥 𝑒 𝑥𝑑𝑥   1. Exponential and Trigonometric   ∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 |
| **Theories/Concepts/Formulas** | **1. Integration by Parts**  From:  *d(uv) = udv + vdu*  Integrating both sides of the equation:  ∫ 𝑑(𝑢𝑣) = ∫ 𝑢𝑑𝑣 + ∫ 𝑣𝑑𝑢  𝑢𝑣 = ∫ 𝑢𝑑𝑣 + ∫ 𝑣𝑑𝑢 |

|  |  |
| --- | --- |
|  | 𝑢𝑣 − ∫ 𝑣𝑑𝑢 = ∫ 𝑢𝑑𝑣  Thus,  ∫ 𝑢𝑑𝑣 = 𝑢𝑣 − ∫ 𝑣𝑑𝑢 |
| In choosing u and dv always remember the following:   1. dx is always included in dv. 2. It must be possible to integrate dv directly in some instances. 3. It is best usually to choose the most complicated factor as dv. |
| **YouTube Link/s** | <https://www.youtube.com/watch?v=bLhxQIdbWW8>  <https://www.youtube.com/watch?v=dqaDSlYdRcs>  <https://www.youtube.com/watch?v=-5Qv7-nfVjI> |
| **Sample Problems** | **Ex. Evaluate the integrals of the following functions:**   1. ∫ 𝑥 𝑒𝑥𝑑𝑥   **Solution**:  u = x dv = ex dx  du = dx v = ex  ∫ 𝑥 𝑒𝑥𝑑𝑥 = 𝑥𝑒𝑥 − ∫ 𝑒𝑥𝑑𝑥  = 𝒙𝒆𝒙 − 𝒆𝒙 + 𝑪 or  = 𝒆𝒙(𝒙 − 𝟏) + 𝑪   1. ∫ 𝑥2 𝑙𝑛𝑥 𝑑𝑥   **Solution**:  u = ln x dv = x2 dx  𝑑𝑥 𝑥3  du = v =  𝑥 3 |

= (𝑙𝑛𝑥)(𝑥

𝒙𝟑 𝒍𝒏𝒙 − (𝒙𝟑) + 𝑪

𝟑

𝟗

𝒙𝟑 (𝒍𝒏𝒙 − 𝟏) + 𝑪

𝟑

𝟑

3

32

) − ∫(𝑥

𝑑𝑥

)( )

= 𝑥

3

3 3 𝑥

## 𝑙𝑛𝑥 − 1 ∫ 𝑥2𝑑𝑥

3

= 𝑥3

3

## 𝑙𝑛𝑥 − 1

𝑥3

(

) + 𝐶

3 3 3

## = or

=

# 3. 𝐴𝑟𝑐𝑡𝑎𝑛 3𝑥 𝑑𝑥

#### Solution:

u = Arctan 3x dv = dx

3𝑑𝑥

du =

9𝑥2+1

v = x

### = (𝐴𝑟𝑐𝑡𝑎𝑛 3𝑥)(𝑥) − ∫(𝑥)

= 𝑥 𝐴𝑟𝑐𝑡𝑎𝑛 3𝑥 − 3 𝑥𝑑𝑥

∫

9𝑥2+1

3𝑑𝑥

( )

2

9𝑥 +1

Let u = 9x2 + 1

1

du = 18xdx, *if*=

18

### = 𝑥 𝐴𝑟𝑐𝑡𝑎𝑛 3𝑥 −

1

### 3 18𝑥𝑑𝑥

18 ∫ 9𝑥2 + 1

= 𝑥 𝐴𝑟𝑐𝑡𝑎𝑛 3𝑥 −

ln(9𝑥2 + 1) + 𝐶

6

=

𝟏

𝒙 𝑨𝒓𝒄𝒕𝒂𝒏 𝟑𝒙 − 𝐥𝐧(𝟗𝒙𝟐 + 𝟏)𝟔 + 𝑪

# 4. ∫ 𝑥2𝑠𝑖𝑛𝑥 𝑑𝑥

#### Solution:

u = x2 dv = sin x dx

du = 2xdx v = -cos x

= (𝑥2)(−𝑐𝑜𝑠𝑥) − ∫(− cos 𝑥) (2𝑥𝑑𝑥)

= −𝑥2 cos 𝑥 + 2 ∫ 𝑥 𝑐𝑜𝑠 𝑥 𝑑𝑥

u = x dv = cos x dx

du = dx v = sin x

= −𝑥2 cos 𝑥 + 2 [𝑥 𝑠𝑖𝑛𝑥 − ∫ 𝑠𝑖𝑛𝑥 𝑑𝑥]

= −𝑥2𝑐𝑜𝑠𝑥 + 2𝑥𝑠𝑖𝑛𝑥 − 2 ∫ 𝑠𝑖𝑛𝑥 𝑑𝑥

= −𝑥2𝑐𝑜𝑠𝑥 + 2𝑥𝑠𝑖𝑛𝑥 + 2𝑐𝑜𝑠𝑥 + 𝐶

𝟐𝒙𝒔𝒊𝒏𝒙 + 𝟐𝒄𝒐𝒔𝒙 − 𝒙𝟐𝒄𝒐𝒔𝒙 + 𝑪

=

# 5. ∫ 𝑒𝑥𝑐𝑜𝑠𝑥 𝑑𝑥

#### Solution:

u = ex dv = cos x dx

du = ex dx v = sin x

∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒 𝑥𝑠𝑖𝑛𝑥 − ∫(𝑠𝑖𝑛𝑥)(𝑒𝑥𝑑𝑥)

∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒 𝑥𝑠𝑖𝑛𝑥 − ∫(𝑒𝑥𝑠𝑖𝑛𝑥 𝑑𝑥)

u = ex dv = sin x dx

du = ex dx v = -cos x

∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒 𝑥𝑠𝑖𝑛𝑥 − [𝑒𝑥(−𝑐𝑜𝑠𝑥)

− ∫(−𝑐𝑜𝑠𝑥)(𝑒𝑥𝑑𝑥)]

∫ 𝒆𝒙𝒄𝒐𝒔𝒙 𝒅𝒙 = 𝒆

𝒙

𝟐

(𝒔𝒊𝒏𝒙 + 𝒄𝒐𝒔𝒙) + 𝑪

∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒 𝑥𝑠𝑖𝑛𝑥 + 𝑒 𝑥𝑐𝑜𝑠𝑥

− ∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥

∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 + ∫ 𝑒 𝑥 𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒 𝑥𝑠𝑖𝑛𝑥

+ 𝑒 𝑥𝑐𝑜𝑠𝑥

2 ∫ 𝑒 𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒 𝑥𝑠𝑖𝑛𝑥 + 𝑒 𝑥𝑐𝑜𝑠𝑥

∫ 𝑒𝑥𝑐𝑜𝑠𝑥 𝑑𝑥 = 𝑒

𝑥

𝑠𝑖𝑛𝑥+𝑒 𝑐𝑜𝑠𝑥 + 𝐶

𝑥

2